The Monomial-Divisor Mirror Map for

Landau-Ginzburg Orbifolds

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ABSTRACT

We present the new explicit geometrical knowledge of the Landau-Ginzburg orbifolds, when a typical type of superpotential is considered. Relying on toric geometry, we show the one-to-one correspondence between some of the (a, c) states with $U(1)$ charges $(-1, 1)$ and the $(1, 1)$ forms coming from blowing-up processes. Consequently, we find the monomial-divisor mirror map for Landau-Ginzburg orbifolds. The possibility of the application of the models of other types is briefly discussed.

$N = 2$ superconformal field theory has attracted the attention in the context of string compactification [1]. Due to its (anti-)chiral ring structure [2, 3] , the theory with $c = 9$ has a Calabi-Yau interpretation, i.e. the (p, q) forms on a Calabi-Yau manifold can be identified with $(3 - p, q)$ states of the (c, c) ring or $(-p, q)$ states of the (a, c) ring, where c (a) stands for (anti-)chiral and the states are labeled by the $U(1)$ charges. These (c, c) and (a, c) rings can be described in terms of the Landau-Ginzburg models.

Recently, the mirror symmetry of Calabi-Yau manifolds has been actively studied [4, 5], since some Yukawa couplings can be determined exactly, assuming that this symmetry is correct. Although this symmetry was first suggested in the $N = 2$ superconformal field theory context [2, 3] , recent analysis is purely geometrical.

Toric geometry gives us the method to examine some of the moduli of Calabi-Yau manifolds. These are the $(1, 1)$ forms coming from the blowing-up process and the $(2, 1)$ forms which are mirror partners of them. These $(2, 1)$ forms can be represented by monomials in a defining equation of a Calabi-Yau manifold. Aspinwall et.al.[6] found that these $(2, 1)$ and $(1, 1)$ forms get interchanged under the mirror map. Hence this mirror map is called "*the monomial-divisor mirror map*".

In this paper, we try to find the corresponding mirror map in the Landau-Ginzburg context. To do this, we first identify the $(-1, 1)$ states with the $(1, 1)$ forms coming from the blowing-up process. Once this identification is made, we can study the geometry of compactified space more deeply in terms of Landau-Ginzburg model.

In this paper, we will restrict our attention to the superpotential of a form $W(X_i) = X_1^{a_1} + X_2^{a_2} + X_3^{a_3} + X_4^{a_4} + X_5^{a_5}$, which corresponds to the Fermat type hypersurface in WCP^4. The Landau-Ginzburg orbifolds are obtained by quotienting with an Abelian symmetry group G of $W(X_i)$, whose element g acts as an $N \times N$ diagonal matrix, $g : X_i \rightarrow e^{2\pi i \tilde{\theta}_i^g} X_i$, where $0 \leq \tilde{\theta}_i^g < 1$. Of course the $U(1)$ twist $j : X_i \rightarrow e^{2\pi i q_i} X_i$ generates the symmetry group of $W(X_i)$, where $q_i = \frac{n_i}{d}$, $W(\lambda^{n_i} X_i) = \lambda^d W(X_i)$ and $\lambda \in \mathbb{C}^*$.

Using the results of Intriligator and Vafa [8], we can construct the (c, c) and (a, c) rings. Also we could have the left and right $U(1)$ charges of the ground state $|h\rangle_{(a,c)}$ in the h-twisted sector of the (a, c) ring. In terms of spectral flow, $|h\rangle_{(a,c)}$ is mapped to the (c,c) state $|h'\rangle_{(c,c)}$ with $h' = hj^{-1}$. Then the charges of the (a,c) ground state of h-twisted sector $|h\rangle_{(a,c)}$ are obtained to be

$$\begin{pmatrix} J_0 \\ \bar{J}_0 \end{pmatrix} |h\rangle_{(a,c)} = \begin{pmatrix} -\sum_{\tilde{\theta}_i^{h'}>0} (1 - q_i - \tilde{\theta}_i^{h'}) + \sum_{\tilde{\theta}_i^{h'}=0} (2q_i - 1) \\ \sum_{\tilde{\theta}_i^{h'}>0} (1 - q_i - \tilde{\theta}_i^{h'}) \end{pmatrix} |h\rangle_{(a,c)}. \qquad (1)$$

Using this result, we see that the $(-1, 1)$ states written in the from $\left|j^l\right\rangle_{(a,c)}$ can always arise from the twisted sector with $I' = 0$, where I' is the number of the invariant fields X_i under the h' action. From the results of ref.[9], we see that the $(2, 1)$ states corresponding to the $(-1, 1)$ states can come from the h' twisted sector with $I' = 0$ or $I' = 2$. So the $(-1, 1)$ states can arise from the twisted sectors with $I' = 2$ only if $\sum_{\tilde{\theta}_i^{h'} = 0} (2q_i - 1) = 0$. This condition implies that the Landau-Ginzburg superpotential contains two trivial fields. So as long as we consider the Landau-Ginzburg models with no or one trivial field, the $(-1, 1)$ states which can be represented by $\left|j^l\right\rangle_{(a,c)}$ may exist only in the twisted sector with $I' = 0$.

Let us turn our attention to geometry. Calabi-Yau manifolds are represented by hypersurfaces in WCP. In general, due to the WCP identification $z_i \sim \lambda^{n_i} z_i, \ \lambda \in \mathbb{C}^*$, we have some fixed sets on a hypersurface. When we consider Calabi-Yau 3-folds, possible fixed sets are fixed points and fixed curves. To obtain a smooth Calabi-Yau manifold we have to blow up these singularities. Hodge numbers $h^{1,1}$ and $h^{2,1}$ change through the blowing-up processes. Especially $h^{1,1}$ increases since new $(1, 1)$ forms arise from exceptional divisors, which come from the resolution of singularities.

Those Calabi-Yau resolutions can be described in terms of toric geometry [10, 11, 5]. Toric geometry describes the structure of a certain class of geometrical spaces in terms of simple combinatorial data. When a space admits a description in terms of toric geometry, many basic and essential characteristics of the space - such as its divisor classes and other aspects of its cohomology - are neatly coded and easily deciphered from the analysis of corresponding lattices. In toric geometry, we are able to deal with some of the exceptional divisors, which we call toric divisors.

First we consider a fixed curve in WCP^4. We will briefly summarize the description of toric divisors in terms of toric data following ref. [10], and explain the idea of the identification in this case.

Let G' be a finite group generated by g' which acts on z_i, homogeneous coordinates of WCP^4, as

$$g' \ : \ [z_1, z_2, z_3, z_4, z_5] \rightarrow [e^{2\pi i x_1} z_1, e^{2\pi i x_2} z_2, z_3, z_4, z_5]. \tag{2}$$

The curve in WCP^4 fixed under the g' action can be written in the form

$$z_3^{a_3} + z_4^{a_4} + z_5^{a_5} = 0, \quad z_1 = z_2 = 0. \tag{3}$$

In this case let us define

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$$\mathbf{n} = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \mid \text{dia} \left[e^{2\pi i x_1}, e^{2\pi i x_2}, \right] \in G' \right\}, \tag{4}$$

$$\triangle = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \mid \sum_{i=1}^{2} x_i = 1, x_i \geq 0 \text{ for all } i \right\}, \tag{5}$$

$$\Gamma = \mathbf{n} \cap \triangle. \tag{6}$$

Γ is a finite subset of the lattice $\mathbf{n}$, and contains the standard base $\{e^i\}_{i=1}^{2}$ of $\mathbb{R}^2$. These are called toric data.

It is known that

$$\left\{ D_\gamma \mid \gamma \in \Gamma - \{e^i\}_{i=1}^{2} \right\}$$
$$= \{\text{toric divisors coming from resolution}\}. \tag{7}$$

In the following we associate a point in the lattice Γ, i.e. a toric divisor, with a $(-1,1)$ state which can be written in the form $\left| j^l \right\rangle_{(a,c)}$ with $I = 3$, if g' can be written in the form j^l, where I is the number of the invariant fields X_i under the j^l action.

We define the phase symmetries ρ_i which act on X_i as

$$\rho_i X_i = e^{2\pi i q_i} X_i, \tag{8}$$

with trivial action for other fields. The operator ρ_i can be represented by a diagonal matrix whose diagonal matrix elements are 1 except for $(\rho_i)_{i,i} = e^{2\pi i q_i}$.

It is obvious that

$$j = \rho_1 \cdots \rho_5. \tag{9}$$

In the j^l-twisted sector, if a field X_i is invariant then

$$\rho_i^l = \rho_i^{l_i} = \text{identity}, \tag{10}$$

where $l_i \equiv l \mod a_i$ and we have

$$j^l = \prod_{l q_i \notin \mathbf{Z}} \rho_i^{l_i}. \tag{11}$$

If the number of i's which satisfy $l q_i \notin \mathbf{Z}$ is 2, i.e. $I = 3$, j^l acts on X_i as

$$j^l \; : \; [X_1, X_2, X_3, X_4, X_5] \sim [e^{2\pi i l_1 q_1} X_1, e^{2\pi i l_2 q_2} X_2, X_3, X_4, X_5]. \tag{12}$$

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with an appropriate renumbering for q_i, if necessary. Then j^l can be equivalent to g' through the identification

$$[e^{2\pi i l_1 q_1}, e^{2\pi i l_2 q_2}] \sim [e^{2\pi i x_1}, e^{2\pi i x_2}].\tag{13}$$

The condition for $\triangle$ is automatically satisfied since $\det j^l = 1$. Thus we can associate a (a,c) state $\left|\prod_{l q_i \notin \mathbf{Z}} \rho_i^{l_i}\right\rangle_{(a,c)}$ with $I{=}3$ with a point in Γ, i.e. a toric divisor.

Further we can calculate the $U(1)$ charges of this state using eq.(1) and the result is

$$(-\sum_{i=1}^{2} l_i q_i, \sum_{i=1}^{2} l_i q_i) \sim (-\sum_{i=1}^{2} x_i, \sum_{i=1}^{2} x_i),\tag{14}$$

through the identification (13). Thus we find that the $U(1)$ charges of the state $\left|\prod_{l q_i \notin \mathbf{Z}} \rho_i^{l_i}\right\rangle_{(a,c)}$ with $I{=}3$ is $(-1, 1)$, since the condition for $\triangle$ holds.

Now we can find the one-to-one correspondence between $(-1, 1)$ states and $(1, 1)$ forms. It is believed that $(-1, 1)$ states correspond to $(1, 1)$ forms and we know a new $(1, 1)$ form comes from a toric divisor during blowing up processes. Therefore, a charge $(-1, 1)$ state $\left|\prod_{l q_i \notin \mathbf{Z}} \rho_i^{l_i}\right\rangle_{(a,c)}$ with $I{=}3$ corresponds to a $(1, 1)$ form coming from a toric divisor through the identification (13).

Also the $(2, 1)$ states can be identified with the $(1, 1)$ forms. By spectral flow, the ground state $|h\rangle_{(a,c)}$ is mapped to the ground state $|h'\rangle_{(c,c)}$ with $h' = hj^{-1}$. So we conclude that the charge $(-1, 1)$ state $|h\rangle_{(a,c)}$ flows to the charge $(2, 1)$ state $|h'\rangle_{(c,c)}$ in h'-twisted sector with $I' = 0$. Thus a $(1, 1)$ form coming from a toric divisor corresponds to a charge $(2,1)$ state $|h'\rangle_{(c,c)}$ in h'-twisted sector with $I' = 0$.

It is easy to see that the state $|j^{-2}\rangle_{(c,c)}$ always satisfies this condition. But the corresponding $(-1, 1)$ state $|j^{-1}\rangle_{(a,c)}$ has $I = 0$. We should associate this state with the origin of the lattice Γ, which corresponds to the $(1, 1)$ form coming from the embedded space WCP^4.

It is instructive to explain the above story by a simple example. Consider the Landau-Ginzburg model with the superpotential

$$W_1 = X_1^8 + X_2^8 + X_3^4 + X_4^4 + X_5^4,\tag{15}$$

with $U(1)$ charges of X_i being

$$(\frac{1}{8}, \frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}).\tag{16}$$

The orbifold model W_1/j has a corresponding $\mathbb{Z}_2$ fixed curve which can be written

$$z_3^4 + z_4^4 + z_5^4 = 0 \quad \text{in } WCP_{(1,1,2,2,2)}^4.\tag{17}$$

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After the blowing up only one (1,1) form comes from a toric divisor.

Using the above discussions we can easily find the state corresponding to this (1,1) form. It is easy to see that in this case the operator which acts like g' in (2) is j^4. So the twisted ground state $|j^4\rangle_{(a,c)}$ is associated with the (1,1) form coming from toric divisor. By flowing to the (c,c) ring, we find that the state $|j^3\rangle_{(c,c)}$ has the same correspondence. It is worth noting that in this model there exists the state $|j^{-1}\rangle_{(a,c)}$ (or $|j^{-2}\rangle_{(c,c)}$) which corresponds to the $(1,1)$ form coming from WCP^4.

Next we consider fixed points in WCP^4. The description of toric divisors in terms of toric data is as follows [10].

Let G' be a finite group generated by g' which acts on z_i as

$$g' \; : \; [z_1, z_2, z_3, z_4, z_5] \rightarrow [e^{2\pi i x_1} z_1, e^{2\pi i x_2} z_2, e^{2\pi i x_3} z_3, z_4, z_5]. \tag{18}$$

The points in WCP^4 fixed under the g' action can be written in the following form

$$z_4^{a_4} + z_5^{a_5} = 0, \quad z_1 = z_2 = z_3 = 0. \tag{19}$$

In this case the toric data are

$$\mathbf{n} = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \mid \mathrm{dia}\left[e^{2\pi i x_1}, e^{2\pi i x_2}, e^{2\pi i x_3} \right] \in G' \right\}, \tag{20}$$

$$\triangle = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \mid \sum_{i=1}^{3} x_i = 1, x_i \geq 0 \text{ for all } i \right\}, \tag{21}$$

$$\Gamma = \mathbf{n} \cap \triangle. \tag{22}$$

Γ is a finite subset of the lattice $\mathbf{n}$, and contains the standard base $\{e^i\}_{i=1}^{3}$ of $\mathbb{R}^3$.

In this case two classes of fixed points are possible. One consists of the isolated fixed points and the other consists of the fixed points on fixed curves. In each type a toric divisor coming from the resolution of fixed point is associated with a point in the lattice

$$\Gamma_{\mathrm{in}} = \mathbf{n} \cap \text{interior} \, (\triangle), \tag{23}$$

which is a sublattice of Γ. In general, the curve is fixed by the subgroup of the group which fixes some points on that curve. This subgroup can be reduced to the group G' in eq.(4), after appropriate renumbering if necessary. So the toric divisors coming from the

resolution of fixed curves on which fixed points sit can be associated with the points in the sublattice of Γ.

For the toric divisors coming from the resolution of fixed curves we can identify a $(1,1)$ form with a $(-1,1)$ state $\left|\prod_{lq_i\notin\mathbf{Z}}\rho_i^{l_i}\right\rangle_{(a,c)}$ with $I=3$ in the same way. Using a similar discussion we can identify a $(1,1)$ form coming from the resolution of fixed point with a $(-1,1)$ state $\left|\prod_{lq_i\notin\mathbf{Z}}\rho_i^{l_i}\right\rangle_{(a,c)}$ with $I=2$.

Let us now turn to the mirror map. It is known that the mirror of Landau-Ginzburg orbifold W/j is obtained to be W/G_m, where G_m is the maximal phase symmetry group of W with determinant 1 [12]. Although the mirror theory W/G_m has the same potential W as the original theory W/j, we will denote it by $\overline{W}$, which consists of the fields $\overline{X_i}$ (of course $\overline{X_i}=X_i$ in this case). This is to make clear which theory we are considering.

Unfortunately, we cannot fully establish mirror pairings of the states, but we can discuss the mirror partners of a special type of states. They are the states which can be written in the form $\left|\prod_i\rho_i^{-l_i}\right\rangle_{(a,c)}$, where l_i are defined mod a_i as before.

By using eq.(1), it can be shown that the left $U(1)$ charge of the state $\left|\rho_i^{-1}\right\rangle_{(a,c)}$ is $-q_i$ and right charge is q_i. This fact suggests that the mirror image of the twisted ground state $\left|\rho_i^{-1}\right\rangle_{(a,c)}$ is $\overline{X_i}|0\rangle$. So we would like to conjecture

$$\left|\rho_i^{-l_i}\right\rangle_{(a,c)} \overset{\text{mirror pair}}{\longleftrightarrow} \overline{X_i}^{l_i}|0\rangle. \tag{24}$$

If we consider the more general twisted ground state $\left|\prod_i\rho_i^{-l_i}\right\rangle_{(a,c)}$, we can write the above mirror pairing as

$$\left|\prod_i\rho_i^{-l_i}\right\rangle_{(a,c)} \overset{\text{mirror pair}}{\longleftrightarrow} \prod_i\overline{X_i}^{l_i}|0\rangle. \tag{25}$$

In terms of this mirror pairing we can find the mirror partner of the $(-1,1)$ state which is discussed above. Since this state can be represented by $\left|\prod_{lq_i\notin\mathbf{Z}}\rho_i^{-l_i}\right\rangle_{(a,c)}$ we see that the mirror partner of this state is $\prod_{lq_i\notin\mathbf{Z}}\overline{X_i}^{l_i}|0\rangle$.

We should call this pairing the monomial-divisor mirror map for Landau-Ginzburg orbifold because this state $\prod_{lq_i\notin\mathbf{Z}}\overline{X_i}^{l_i}|0\rangle$ must correspond to the monomial $\prod_{lq_i\notin\mathbf{Z}}X_i^{l_i}$ which survives the orbifoldization by G_m, where we have omitted the bar over X_i. The monomial-divisor mirror map for Calabi-Yau mirror pair is studied in [5, 13], and we have checked that our results exactly correspond to the results obtained therein .

For example, we take the superpotential (15) again (this model is considered in [5]). In this example, the twisted ground state $|j^4\rangle_{(a,c)}$ is associated with the $(1,1)$ form coming

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from resolution. Since $j^8 = 1$ we have

$$\left| j^4 \right\rangle_{(a,c)} \sim \left| j^{-4} \right\rangle_{(a,c)}. \tag{26}$$

Through the fact

$$j^{-4} = \rho_1^{-4} \rho_2^{-4}, \tag{27}$$

we find the mirror pairing

$$\left| \rho_1^{-4} \rho_2^{-4} \right\rangle_{(a,c)} \overset{\text{mirror pair}}{\longleftrightarrow} \overline{X_1}^4 \overline{X_2}^4 \left| 0 \right\rangle. \tag{28}$$

So we conclude that the monomial which survives after the orbifoldization by the G_m action is $X_1^4 X_2^4$, where we have omitted the bar over X_i.

We can find the mirror partner of the state $\left| j^{-1} \right\rangle_{(a,c)}$ which corresponds to the $(1,1)$ form coming from WCP^4. The mirror image is $\overline{X_1 X_2 X_3 X_4 X_5} \left| 0 \right\rangle$. This corresponds to the monomial $X_1 X_2 X_3 X_4 X_5$ which is evidently invariant under the G_m action. We summarize these results in Table 1 together with those for (c,c) states.

(c,c) state	(a,c) state	mirror partner
$\left| j^{-2} \right\rangle_{(c,c)}$	$\left| j^{-1} \right\rangle_{(a,c)}$	$X_1 X_2 X_3 X_4 X_5 \left| 0 \right\rangle$
$\left| j^{-4} \right\rangle_{(c,c)}$	$\left| j^{-3} \right\rangle_{(a,c)}$	$X_1^4 X_2^4 \left| 0 \right\rangle$

Table 1: The monomial-divisor mirror map for Landau-Ginzburg orbifolds of W_1

As a more complicated example we take the following Landau-Ginzburg superpotential

$$W_2 = X_1^3 + X_2^3 + X_3^6 + X_4^9 + X_5^{18}, \tag{29}$$

with $U(1)$ charges

$$(\frac{1}{3}, \frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{18}). \tag{30}$$

This model is considered in [13, 14]. The orbifold model W_2/j has one corresponding $\mathbb{Z}_2$ fixed curve, one corresponding $\mathbb{Z}_3$ fixed curve and corresponding $\mathbb{Z}_6$ fixed points on the intersections of these curves. They can be written as

$$\mathbb{Z}_2 \text{ fixed curve } z_1^3 + z_2^3 + z_4^9 = 0 \tag{31}$$

$$\mathbb{Z}_3 \text{ fixed curve } z_1^3 + z_2^3 + z_3^6 = 0 \tag{32}$$

$$\mathbb{Z}_6 \text{ fixed points } z_1^3 + z_2^3 = 0 \quad \text{in } WCP^4_{(6,6,3,2,1)}. \tag{33}$$

It is easy to find the states which correspond to the (1,1) forms coming from toric divisors and their mirror partners. The results are displayed in Table 2, where we have omitted the bar over X_i.

(c,c) state	(a,c) state	mirror partner
$\left\lvert j^{-2} \right\rangle_{(c,c)}$	$\left\lvert j^{-1} \right\rangle_{(a,c)}$	$X_1 X_2 X_3 X_4 X_5 \left\lvert 0 \right\rangle$
$\left\lvert j^{-4} \right\rangle_{(c,c)}$	$\left\lvert j^{-3} \right\rangle_{(a,c)}$	$X_3{}^3 X_4{}^3 X_5{}^3 \left\lvert 0 \right\rangle$
$\left\lvert j^{-7} \right\rangle_{(c,c)}$	$\left\lvert j^{-6} \right\rangle_{(a,c)}$	$X_4{}^6 X_5{}^6 \left\lvert 0 \right\rangle$
$\left\lvert j^{-10} \right\rangle_{(c,c)}$	$\left\lvert j^{-9} \right\rangle_{(a,c)}$	$X_3{}^3 X_5{}^9 \left\lvert 0 \right\rangle$
$\left\lvert j^{-13} \right\rangle_{(c,c)}$	$\left\lvert j^{-12} \right\rangle_{(a,c)}$	$X_4{}^3 X_5{}^{12} \left\lvert 0 \right\rangle$

Table 2: The monomial-divisor mirror map for Landau-Ginzburg orbifolds of W_2

This result agrees with the one obtained in [13] . Note that we do not need any geometrical informations such as the number of fixed sets or the relations among them.

The corresponding Calabi-Yau manifold has five $(1,1)$ forms discussed above and two $(1,1)$ forms whose mirror partners cannot be described by the monomials. The $(-1,1)$ states corresponding to these two $(1,1)$ forms are represented by $X_1 \lvert j^{-2} \rangle_{(a,c)}$ and $X_2 \lvert j^{-2} \rangle_{(a,c)}$. But we do not know the one-to-one correspondence and their mirror partners.

Let us discuss the meaning of our results a little bit more. We used the toric data to identify (a,c) states with $(1,1)$ forms. This is a remarkable fact. At this moment, it is unclear why Landau-Ginzburg models have Calabi-Yau interpretations. However, our method could partially answer to this problem. The toric data are essential and they have two different interpretations, i.e. cohomologies on a Calabi-Yau manifold and (a,c) states in a Landau-Ginzburg orbifolds.

Also, our method could answer to another important problem, i.e. why strings do not feel singularities. Our analyses show that the states coming from twisted sectors correspond to the forms coming from blowing-up processes. Since the modular invariance of the Witten index requires these twisted sectors, we obtain the index as an Euler number of a smooth Calabi-Yau manifold.

The superpotential considered in this paper corresponds to the Gepner model of A-type [1]. So our analysis will give the insight into the understanding of the exact mirror map at the level of the conformal field theory.

Some problems still remain. In this paper we restrict our attention to the Fermat-type potential with five fields. Of course the potentials of other types are possible for string compactification. For those potentials, there could be new singularities whose resolutions cannot be described in terms of toric geometry. The $(-1, 1)$ states, which correspond to the $(1, 1)$ forms coming from resolutions of these singularities of new type, should not be written in the form $\left| j^l \right\rangle_{(a,c)}$. However, in general there can be $(-1, 1)$ states written in the form $\prod_{l q_i \in \mathbb{Z}} X_i^{l_i} \left| j^l \right\rangle_{(a,c)}$ with at least one $l_i > 0$. From the results of ref.[9], we see that corresponding $(2, 1)$ states can arise from the j^{l-1}-twisted sector with $I' = 2$. In ref.[15], non-Fermat-type potentials and their mirror maps are considered. But we can not fully establish the one-to-one correspondence between the $(-1, 1)$ states and the $(1, 1)$ forms.

However, there are non-Fermat-type potentials with only the singularities which can be treated through toric geometry. For example, there is the hypersurface embedded in $WCP^4_{(1,2,2,2,3)}$, which gets two toric divisors after blowing up [16]. For this model, our identification of the $(-1, 1)$ states holds and we have checked that the monomial-divisor mirror map for Landau-Ginzburg orbifold is indeed realized.

Although we do not know an exact method for the calculation of the Yukawa couplings in the framework of Landau-Ginzburg models, our analysis will be useful to study the moduli dependence of the Yukawa couplings.

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